

RESPONSE RATE AND CHANGEOVER PERFORMANCE ON CONCURRENT VARIABLE-INTERVAL SCHEDULES¹

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Six pigeons were exposed to variable-interval schedules arranged on one, two, three, and four response keys. The reinforcement rate was also varied across conditions. Numbers of responses, the time spent responding, the number of reinforcements, and the number of changeovers between keys were recorded. Response rates on each key were an increasing function of reinforcement rate on that key and a decreasing function of the reinforcement rate on other keys. Response and time-allocation ratios under-matched ratios of obtained reinforcements. Three sets of equations were developed to express changeover rate as a function of response rate, time allocation, and reinforcement rate respectively. These functions were then applied to a broad range of experiments in the literature in order to test their generality. Further expressions were developed to account for changeover rates reported in experiments where changeover delays were varied.

Key words: variable-interval schedules, concurrent schedules, changeovers, response rate, preference, pecking, pigeons

DEFINITION OF SYMBOLS

Let i, j, k, l, \dots, n denote different keys (schedules or alternatives).

Let n denote the total number of keys.

Let N_i denote the number of responses on key i . Therefore, total number of responses

$$(\Sigma N) = \sum_i^n N_i$$

Let T_i denote the time allocated to responding on key i . Therefore, total time

$$(\Sigma T) = \sum_i^n T_i$$

Let N_{ij} denote the number of changeovers from key i to key j . That is, the number of pairs of responses in which a response on key i is followed by a response on key j .

Let P_i denote the overall response rate on key i where

$$P_i = \frac{N_i}{\Sigma T}$$

Therefore, the total response rate $(\Sigma P) = \sum_i^n P_i$.

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Let R_i denote the overall obtained reinforcement rate resulting from responses on key i , where R_i is the number of reinforcements obtained on key i divided by ΣT . Therefore,

$$\text{total reinforcement rate } (\Sigma R) = \sum_i^n R_i$$

Let C_{ij} denote the changeover rate from key i to key j , where

$$C_{ij} = \frac{N_{ij}}{\Sigma T}$$

Therefore, total changeover rate $(\Sigma C) = \sum_{i,j}^n C_{ij}$.

Let C_i denote the changeover rate from key i to all other keys.

Therefore, $C_i = \sum_j^n C_{ij}$ and $\Sigma C = \sum_i^n C_i$.

During the past 15 yr, two-schedule concurrent variable-interval schedule performance has been investigated by many researchers (Baum 1974b; Davison and Hunter, 1976; Herrnstein, 1961). Such performance has periodically been reviewed in detail (Baum, 1974b; de Villiers, 1977; Herrnstein, 1970). The concurrent variable-interval data appear well described by the following relations (Baum, 1974b, Davison and Hunter, 1976),

$$\frac{P_i}{P_j} = c_1 \left[\frac{R_i}{R_j} \right]^{a_1} \quad (1)$$

$$\frac{T_i}{T_j} = c_2 \left[\frac{R_i}{R_j} \right]^{a_2} \quad (2)$$

where P_i and P_j are the response rates, T_i and T_j the times allocated, and R_i and R_j the reinforcement rates obtained on response manipulanda (keys) i and j ; the parameters c_i and c_j are referred to as the bias between responses or times (Baum, 1974b; Davison and Hunter, 1976), and the exponents a_i and a_j are measures of the sensitivity with which performance changes with changes in reinforcement rate (Davison and Hunter, 1976).

Davison and Hunter (1976) showed that Equations 1 and 2 described three-key concurrent variable-interval performance as well as two-key performance. They also showed that the addition of the third key had no effect on the values of a or c .

Herrnstein (1970, 1974) has proposed Equation 3 to account for absolute response rates:

$$P_i = k \frac{R_i}{\sum_{i=0} R} \quad (3)$$

where R is the total obtained reinforcement rate and R_0 is the reinforcement rate for all behavior other than the defined operants. Equation 3 predicts Equation 1 only when $a = 1$. The exponent a is, however, usually less than one (Davison and Hunter, 1976; Lobb and Davison, 1975; Myers and Myers, 1977) and the corresponding performance is called undermatching.

In order to achieve a function that predicted absolute response rates and which, unlike Equation 3, also implied Equation 1, Davison and Hunter (1976) proposed Equations 4 and 5 to account for absolute response rate:

$$P_i = k_i \left[\frac{R_i}{\sum R} \right]^{a_i} \quad (4)$$

$$P_i = k_i \frac{R_i^{a_i}}{\sum (R_i^{a_i})} \quad (5)$$

They found that the exponent a_i in Equation 4 was consistently less than one and that k_i was relatively constant, compared with k in Equation 3. It should be noted here that $c_i = \frac{k_i}{k_j}$, and that Equations 4 and 5 both imply Equation 1 and that $a_3 = a_1$. Although Equations 4 and 5 account for a large proportion of the data variance, they are nonlinear equations, and consequently present theoretical problems. If the response rates are added (ΣP), then this value when introduced into Equation 4 is given by,

$$\Sigma P = k \left[\frac{\Sigma R}{\Sigma R} \right]^a = k. \quad (4a)$$

However, the mathematical summation implies that

$$\Sigma P = k \frac{\Sigma (R^a)}{(\Sigma R)^a} \neq k \text{ if } a \neq 1. \quad (4b)$$

Similarly, Equation 5 mathematically implies that

$$\Sigma P = k \frac{\Sigma (R^a)}{\Sigma (R^a)} = k, \quad (5a)$$

but when ΣP is introduced into Equation 5 the implication is that,

$$\Sigma P = k \frac{(\Sigma R)^a}{\Sigma (R^a)} \neq k \text{ when } a \neq 1. \quad (5b)$$

However, as simple descriptors of performance, Equations 4 and 5 are the best available at present, and thus they are both utilized in the present report.

In all but four of the experiments reported to date, subjects have had to allocate responses and time between only two alternatives (keys). Reynolds (1963), Davison and Hunter (1976), and Pliskoff and Brown (1976) have investigated performance among three alternatives; Miller and Loveland (1975) studied five-key performance in a single condition. One of the purposes of the present study was to extend the application of Equations 1, 2, 4, and 5 to four-key concurrent schedules.

Aside from response rate and time allocation, a further dependent variable, which has been recorded more often than discussed, is the frequency with which the subject changes over (or switches) from responding on one alternative to responding on another. Although the relation between changeover rate and reinforcement rate has not been formally quantified, Catania (1963) and Herrnstein (1961) have reported that changeover rate decreases with more dissimilar reinforcement rates.

Changeover rate has been plotted against reinforcement rate in at least three different ways. The formulae for these plots are:

- (1) The difference relation: Herrnstein (1961) plotted total changeover rate (ΣC) against the absolute difference in the proportions of reinforcement delivered by the two schedules.

That is

$$\Sigma C \propto \left| \frac{R_1 - R_2}{R_1 + R_2} \right|. \quad (6)$$

It is not clear how to make this relation a linear equation. For instance if $R_1 = R_2$, the right side of Equation 6 is zero, but the changeover rate on the left side is likely to be maximal (Herrnstein, 1961). There appears to be no theoretical basis to this relation. Equation 6 also predicts the unlikely result that two equal schedules providing 100 reinforcements per hour will result in the same changeover rate as two equal schedules providing one reinforcement every 10 hr.

- (2) The relative equation: Miller and Loveland (1975) plotted relative changeover rate ($C_1/\Sigma C$) against relative reinforcement rate. That is:

$$\frac{C_1}{\Sigma C} = \frac{R_1}{\Sigma R}. \quad (7)$$

This equation has a logical flaw when applied to the two-key situation. It predicts that the changeover proportion in a two-key experiment will equal the reinforcement-rate proportion. However, with only two keys, the changeover proportion must always be 0.5, whereas the reinforcement-rate proportion is free to vary from zero to 1.0. This equation cannot be used to predict absolute changeover rates.

- (3) The ratio equations: Baum (1975, 1976) plotted changeover rate (ΣC) against reinforcement-rate ratio (R_1/R_2), or response-rate ratio (P_1/P_2), or time-allocation ratio (T_1/T_2), that is:

$$\Sigma C = f(R_1/R_2) \quad (8)$$

$$\Sigma C = f(P_1/P_2) \quad (9)$$

$$\Sigma C = f(T_1/T_2). \quad (10)$$

Stubbs, Pliskoff, and Reid (1977) plotted the interchangeover times (T_1/N_{12} and T_2/N_{21}) against the reinforcement ratios (R_1/R_2 and R_2/R_1) as a power function. That is

$$\frac{T_1}{N_{12}} = K \left[\frac{R_1}{R_2} \right]^a \quad (11)$$

$$\frac{T_2}{N_{21}} = K \left[\frac{R_2}{R_1} \right]^a. \quad (12)$$

Theoretical justifications have not been given for any of these models, nor is it clear how these models might be applied to account for changeover performance among three or more alternatives.

The aims of the present paper may be summarized as follows:

- (a) to extend more fully the analysis of concurrent response rate performance to more than three keys.
- (b) to find a model appropriate for describing changeover performance when 2, 3, or 4 keys are available.
- (c) to apply this model to a broad range of experiments in the literature, including those where changeover delay (COD) was varied.

METHOD

Subjects

Six homing pigeons, numbered H1 to H6, which had previously served in a discrete-trial auditory discrimination experiment, were individually housed with an ambient air temperature of 18°C and maintained at 80% of their free-feeding body weights.

Apparatus

Conventional relay equipment, situated in a room separate from the experimental chamber, controlled all experimental events. All data (reinforcements per key, responses per key, time per key, and changeovers between keys) were recorded on 24 impulse counters. All the timers were initially calibrated, and were periodically checked with a Marconi Instruments counter timer (model TF 2418). Because the pigeon could change keys during the 50-msec pulse of the time counter, the total session time was recorded on a separate high-speed counter and compared with the sum of the recorded key times. The difference between the recorded total session time and the sum of the individually recorded key times was never more than 0.1%.

The sound-attenuated experimental chamber was 40 cm high, 30 cm deep and 35 cm across and was fitted with an exhaust fan, which helped mask external noise. It contained four response keys numbered from left to right 1, 2, 3, and 4. The keys were 7.5 cm apart, and 24 cm from the wire-mesh floor. Each key could be illuminated achromatically. In addition to the tactile and auditory feedback resulting from the key itself, feedback for pecks with forces exceeding approximately 0.1 N was arranged by way of an offset of all keylights for the duration of the key peck. No illumination was provided in the chamber apart from the white magazine and keylights. Pecks on darkened keys always were ineffec-

tive. A grain hopper was situated midway between the two centre keys and 7 cm from the floor. During reinforcement, the keylights were extinguished and the hopper illuminated.

The reinforcer consisted of a 3.9-sec access to the hopper containing wheat, and sessions were terminated in blackout after a fixed number of reinforcements had been obtained. This number (Table 2) was 50 or less, and gave a session time of not more than 50 min. Sessions were conducted seven days a week and supplementary feeding of mixed grain was given, if required, immediately after daily training sessions.

Procedure

Since all pigeons had experience with a three-key discrimination procedure, no key-peck training was necessary and the subjects were placed directly on a concurrent VI 120-sec VI 240-sec schedule without a COD. After seven sessions, a 0.2-sec COD between all keys was introduced for four sessions. This value was next changed to 0.5 sec for five sessions and then to 1.5 sec for the remainder of the experiment. The VI schedules associated with the keys were completely independent of each other and each consisted of a randomized sequence of 12 intervals taken from the arithmetic progression

$$\frac{x_{i=0}^n(a + ib)}{60},$$

where *x* is the value in seconds of the arithmetic mean of the VI schedule intervals. The

parameters *n*, *a*, and *b*, were fixed at 11, 5, and 10 sec respectively in the present experiment.

Experimental conditions were terminated when each pigeon had reached a defined stability criterion five, not necessarily consecutive, times. This criterion was that the median of each of the relative response measures *N*₁/*N*₁ + *N*₂, *N*₂/*N*₂ + *N*₃, *N*₃/*N*₃ + *N*₄, and *N*₄/*N*₄ + *N*₁ in the last five sessions did not differ by more than 0.05 from the median of the five sessions before these. When all six subjects had obtained this criterion five times for the relative response measures, the experimental condition was changed for all pigeons simultaneously. The numbers of sessions per condition are shown in Table 1.

In Conditions 1, 2, 4, and 8, the schedules on Keys 2 and 3 were held constant at VI 120-sec and VI 240-sec respectively as other schedules were added to the previously dark and inoperative Keys 1 and 4 (Table 1). In Conditions 3 to 7 and 12, the schedules on Keys, 1, 2, and 3 were held constant at VI 480-sec, VI 120-sec, and VI 240-sec respectively, as the schedule on Key 4 was varied randomly across conditions from 7.5 to 240 reinforcements per hour. In Conditions 4, 9, 10, and 11 the total arranged reinforcement rate was held constant at 112.5, as the number of keys providing reinforcement was reduced across conditions from four keys down to a single key respectively.

In all conditions, the number of responses (*N*₁), the time spent on the operative keys (*T*₁), and the reinforcements obtained, were re-

Table 1

Sequence of conditions, variable-interval (VI) mean values in seconds on each key, total scheduled reinforcement rate in reinforcements per hour (rft/hr), and number of sessions per condition, for all six pigeons. Extn denotes extinction.

Condition	VI Schedules (seconds)				Total Scheduled Reinforcement Rate (rft/hr)	Number of Sessions per Condition
	Key 1	Key 2	Key 3	Key 4		
1	off	120	240	off	45	31
2	480	120	240	off	52.5	35
3	480	120	240	360	62.5	32
4	480	120	240	60	112.5	46
5	480	120	240	180	72.5	32
6	480	120	240	30	172.5	28
7	480	120	240	480	60	34
8	Extn	120	240	480	52.5	25
9	Extn	112	56	224	112.5	31
10	Extn	48	96	Extn	112.5	28
11	Extn	32	Extn	Extn	112.5	22
12	480	120	240	15	292.5	34

Table 2

Sequence of experimental conditions, number of responses, time spent, and number of reinforcements obtained on each key. Shown in parentheses are the scheduled reinforcement rates on each key, with Keys 1, 2, 3, and 4 reading downward. The data shown are the sum, and below this in brackets the standard deviation, of the last five experimental sessions of the condition.

(Rft/h) Condition	Bird	Responses				Time (seconds)				Reinforcements Obtained			
		Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4
1													
(off)	H1	0	7426 (285)	5095 (135)	0	0	11363 (135)	4848 (119)	0	0	135 (0)	65 (0)	0
(30)	H2	0	10828 (132)	3778 (304)	0	0	11064 (355)	4610 (128)	0	0	138 (2)	62 (2)	0
(15)	H3	0	17114 (558)	9252 (453)	0	0	10231 (178)	5765 (405)	0	0	133 (1)	67 (1)	0
(off)	H4	0	14603 (234)	6010 (114)	0	0	11937 (101)	4082 (171)	0	0	135 (1)	65 (1)	0
	H5	0	15008 (136)	3823 (310)	0	0	13258 (116)	2239 (172)	0	0	141 (1)	59 (1)	0
	H6	0	10861 (234)	5238 (238)	0	0	10477 (369)	5400 (157)	0	0	137 (1)	63 (1)	0
2													
(7.5)	H1	2420 (136)	8123 (246)	3902 (215)	0	1885 (112)	7143 (138)	3761 (158)	0	26 (1)	98 (1)	51 (1)	0
(30)	H2	1998 (98)	7502 (343)	5997 (299)	0	1240 (44)	5714 (156)	6007 (118)	0	23 (1)	98 (2)	54 (1)	0
(15)	H3	3654 (296)	10844 (160)	5790 (160)	0	2022 (144)	6356 (136)	4229 (87)	0	26 (1)	97 (2)	52 (1)	0
(off)	H4	2444 (235)	7403 (151)	2627 (157)	0	1723 (178)	8425 (149)	1959 (105)	0	26 (1)	102 (1)	47 (1)	0
	H5	646 (24)	12258 (140)	5847 (89)	0	593 (25)	7844 (46)	4193 (48)	0	22 (1)	99 (1)	54 (1)	0
	H6	2657 (65)	8959 (200)	4991 (241)	0	1838 (43)	6080 (146)	4114 (144)	0	24 (1)	95 (1)	56 (1)	0
3													
(7.5)	H1	1488 (60)	4945 (66)	2529 (69)	1455 (41)	1328 (45)	5934 (52)	2438 (100)	1530 (73)	20 (1)	85 (2)	41 (1)	29
(30)	H2	1617 (57)	5997 (196)	3262 (146)	2276 (146)	1386 (57)	4881 (137)	2530 (98)	2110 (75)	19 (1)	85 (1)	44 (0)	27
(15)	H3	1902 (137)	6855 (197)	4727 (278)	1512 (144)	1318 (70)	4550 (154)	3290 (170)	1144 (103)	25 (1)	85 (1)	42 (1)	23
(10)	H4	1655 (192)	8502 (482)	2727 (370)	1625 (172)	1088 (52)	7390 (246)	1510 (44)	919 (38)	19 (1)	86 (1)	41 (1)	29
	H5	1611 (58)	8528 (357)	4038 (91)	1594 (44)	1309 (52)	5011 (200)	2949 (39)	1216 (56)	19 (1)	86 (1)	44 (0)	26
	H6	2527 (126)	7825 (94)	3458 (114)	2789 (213)	1717 (104)	4545 (54)	2649 (103)	1688 (110)	24 (1)	81 (1)	41 (1)	29

Table 2—continued

Condition (Rft/h)	Bird	Responses				Time (seconds)				Reinforcements Obtained			
		Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4
4	H1	874 (43)	1031 (46)	660 (29)	1874 (67)	860 (44)	1318 (47)	802 (22)	3118 (81)	12 (1)	44 (1)	24 (1)	95 (2)
	H2	841 (26)	3925 (49)	1092 (75)	4351 (74)	659 (13)	1898 (34)	693 (33)	2747 (35)	18 (1)	43 (1)	21 (2)	93 (2)
	H3	766 (48)	517 (27)	364 (21)	3577 (130)	681 (26)	452 (16)	305 (9)	5040 (218)	25 (2)	33 (1)	17 (2)	100 (3)
	H4	732 (41)	2689 (142)	523 (14)	2839 (97)	602 (33)	1784 (58)	422 (17)	3355 (67)	17 (2)	45 (1)	20 (1)	93 (2)
	H5	862 (14)	2443 (38)	1196 (36)	3519 (89)	613 (17)	1551 (22)	799 (12)	3012 (53)	14 (1)	45 (1)	26 (1)	90 (1)
	H6	1155 (61)	2644 (70)	723 (49)	4208 (106)	797 (33)	1798 (72)	505 (29)	3292 (81)	15 (1)	48 (1)	18 (1)	94 (1)
5	H1	1011 (40)	2562 (66)	1107 (31)	1824 (65)	1430 (55)	3987 (90)	1468 (22)	2680 (85)	20 (1)	68 (1)	36 (1)	51 (0)
	H2	676 (9)	6174 (61)	3235 (81)	3452 (44)	593 (7)	4281 (44)	2034 (43)	2410 (30)	15 (1)	70 (2)	38 (2)	52 (1)
	H3	2507 (107)	5442 (188)	3069 (108)	3082 (78)	1488 (61)	3430 (116)	1933 (54)	2541 (61)	20 (1)	71 (2)	36 (2)	48 (1)
	H4	1182 (77)	7118 (187)	1209 (49)	3404 (145)	960 (56)	5233 (137)	909 (41)	2602 (89)	20 (1)	77 (1)	34 (2)	44 (1)
	H5	1376 (43)	6296 (184)	2397 (59)	3232 (59)	1254 (18)	3777 (79)	1644 (37)	2810 (47)	17 (1)	70 (1)	37 (1)	51 (1)
	H6	2082 (60)	5201 (128)	2797 (72)	4662 (104)	1547 (46)	3325 (76)	1634 (36)	2804 (52)	18 (1)	71 (2)	36 (1)	50 (1)
6	H1	330 (9)	667 (30)	272 (14)	2072 (38)	314 (6)	925 (28)	407 (18)	2892 (29)	6 (0)	34 (0)	14 (1)	121 (1)
	H2	501 (20)	2191 (39)	811 (22)	3081 (68)	376 (14)	1475 (23)	543 (9)	2125 (22)	8 (1)	34 (1)	16 (1)	117 (1)
	H3	365 (18)	852 (57)	550 (21)	2780 (52)	343 (15)	620 (41)	379 (15)	2904 (43)	10 (1)	29 (1)	16 (1)	120 (1)
	H4	505 (13)	1940 (153)	315 (7)	2862 (116)	352 (17)	1282 (98)	284 (6)	2624 (91)	8 (1)	33 (1)	15 (0)	119 (0)
	H5	330 (25)	1151 (58)	393 (14)	1237 (21)	392 (20)	988 (34)	690 (9)	2444 (40)	11 (0)	33 (1)	17 (1)	114 (1)
	H6	626 (26)	1370 (57)	591 (19)	3184 (74)	432 (18)	1019 (38)	423 (12)	2624 (41)	6 (0)	33 (1)	17 (1)	119 (1)
7	H1	1092 (48)	5183 (81)	2067 (73)	1332 (82)	1327 (48)	6207 (123)	2339 (82)	1362 (78)	21 (1)	88 (1)	42 (1)	24 (1)
	H2	2602 (75)	8598 (81)	3840 (73)	1664 (82)	2071 (48)	5325 (123)	2280 (82)	1215 (78)	22 (1)	88 (1)	44 (1)	21 (1)

Table 2—continued

Condition (Rft/h)	Bird	Responses				Time (seconds)				Reinforcements Obtained			
		Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4
(30)	H3	(91) 2660	(94) 6587	(128) 3529	(85) 1177	(55) 2189	(48) 5142	(59) 2914	(62) 1167	(1) 22	(1) 91	(0) 44	(1) 18
(15)	H4	(144) 1298	(351) 9355	(193) 2822	(52) 1906	(138) 1113	(263) 7256	(137) 1656	(40) 1234	(1) 22	(2) 90	(1) 41	(1) 22
(7.5)	H5	(38) 962	(171) 9204	(130) 2977	(126) 734	(44) 1158	(139) 5558	(63) 2555	(73) 1333	(1) 22	(2) 84	(1) 45	(1) 24
	H6	(41) 2768	(144) 7630	(148) 4297	(16) 1873	(44) 2322	(62) 4798	(89) 2786	(19) 1276	(1) 22	(1) 89	(1) 45	(1) 19
		(85)	(169)	(227)	(94)	(85)	(106)	(110)	(47)	(1)	(1)	(1)	(1)
8	H1	3	6573	3046	1425	4	8161	2934	1368	0	101	51	23
(Extn)	H2	(1) 244	(123) 8978	(71) 5634	(41) 2480	(1) 139	(116) 6496	(54) 4158	(43) 1667	(1) 0	(1) 102	(1) 47	(1) 26
(30)	H3	(19) 16	(152) 9094	(120) 5170	(76) 2434	(11) 19	(94) 7923	(98) 3481	(44) 2051	(1) 0	(1) 100	(1) 49	(1) 26
(15)	H4	(2) 2	(454) 12247	(109) 3133	(122) 2448	(3) 0	(232) 9338	(70) 1949	(88) 1386	(1) 0	(1) 101	(1) 50	(1) 24
(7.5)	H5	(1) 74	(254) 9893	(18) 5102	(80) 1078	69	(115) 6717	(28) 4402	(47) 1498	(1) 0	(1) 103	(1) 51	(1) 21
	H6	(15) 83	(175) 11663	(79) 6077	(54) 2677	(13) 62	(54) 8020	(140) 3658	(62) 1518	(1) 0	(1) 98	(1) 53	(1) 24
		(12)	(375)	(298)	(75)	(8)	(184)	(148)	(42)	(1)	(1)	(1)	(1)
9	H1	0	1258	2827	925	0	1836	3368	1111	0	48	101	26
(Extn)	H2	(0) 47	(23) 2928	(51) 3634	(14) 2152	30	(24) 1966	(37) 2687	(17) 1404	(1) 0	(1) 52	(0) 94	(0) 29
(32.14)	H3	(13) 0	(51) 2405	(78) 6029	(67) 813	(8) 0	(33) 1788	(35) 4031	(34) 582	(1) 0	(1) 49	(1) 108	(1) 18
(64.29)	H4	(0) 0	(55) 4604	(100) 2212	(30) 1124	0	(53) 3486	(72) 1963	(17) 768	(1) 0	(0) 54	(1) 95	(1) 26
(16.07)	H5	(0) 0	(56) 2769	(70) 3007	(51) 387	0	(24) 1586	(36) 3792	(25) 797	(1) 0	(1) 53	(1) 99	(1) 23
	H6	(0) 0	(43) 2268	(47) 3640	(13) 1095	0	(19) 2157	(36) 3355	(26) 781	(1) 0	(1) 47	(1) 105	(1) 23
		(0)	(53)	(134)	(55)		(63)	(78)	(17)	(1)	(1)	(1)	(1)
10	H1	3	2461	1885	0	0	4122	2036	0	0	118	57	0
(Extn)	H2	(1) 24	(26) 4578	(47) 2850	31	20	(22) 3629	(46) 2290	(17) 23	(1) 0	(1) 115	(1) 60	(1) 0
(75)	H3	(3) 0	(45) 5600	(22) 3356	(6) 0	(3) 0	(22) 3679	(29) 1990	(4) 0	(0) 0	(0) 119	(0) 56	(0) 0
(37.5)			(59)	(69)			(20)	(35)			(0)	(0)	

Table 2--continued

Condition (Rft/h)	Bird	Time (seconds)				Reinforcements Obtained							
		Key 1	Key 2	Key 3	Key 4	Key 1	Key 2	Key 3	Key 4				
11	H4	0	4975 (71)	1934 (35)	14 (5)	0	4322 (56)	2098 (257)	9 (2)	0	119 (0)	56 (0)	0
	H5	0	5388 (92)	1737 (44)	96 (2)	0	3882 (53)	2208 (46)	153 (6)	0	119 (0)	56 (0)	0
	H6	12 (2)	3824 (111)	2223 (42)	2 (1)	14 (3)	3888 (52)	2225 (49)	3 (1)	0	120 (1)	55 (1)	0
	H1	0	3731 (74)	336 (10)	0	0	5912 (18)	279 (10)	4 (2)	0	175 (0)	0 (0)	0
	H2	0	6974 (102)	73 (12)	34 (8)	0	6097 (12)	46 (7)	29 (6)	0	175 (0)	0 (0)	0
	H3	2 (1)	6184 (86)	37 (5)	0	1 (0)	6093 (26)	31 (5)	0 (0)	0	175 (0)	0 (0)	0
12	H4	0	7985 (67)	2 (1)	0	0	6155 (11)	3 (1)	0 (0)	0	175 (0)	0 (0)	0
	H5	14 (1)	7263 (151)	3 (1)	1 (0)	13 (2)	6167 (14)	2 (1)	5 (2)	0	175 (0)	0 (0)	0
	H6	0	7511 (91)	48 (5)	25 (4)	0	6088 (12)	55 (4)	18 (2)	0	175 (0)	0 (0)	0
	H1	102 (7)	427 (16)	257 (16)	1674 (34)	132 (8)	491 (9)	235 (10)	1917 (29)	6 (1)	22 (1)	12 (1)	135 (1)
	H2	0	1338 (13)	197 (9)	2517 (22)	4	947 (16)	155 (8)	1966 (19)	0	19 (0)	11 (1)	145 (1)
	H3	179 (10)	568 (31)	199 (15)	1326 (51)	166 (5)	495 (21)	175 (12)	2092 (44)	8 (1)	20 (1)	7 (1)	140 (2)
15	H4	38 (14)	862 (24)	217 (22)	2055 (43)	23 (7)	652 (21)	167 (16)	2094 (37)	1 (0)	23 (1)	8 (1)	143 (1)
	H5	188 (9)	473 (44)	233 (10)	1147 (44)	196 (7)	392 (26)	260 (7)	2230 (39)	8 (1)	16 (0)	8 (1)	143 (1)
	H6	1	662 (38)	289 (11)	1939 (32)	8	563 (23)	251 (8)	2073 (19)	0	21 (1)	12 (1)	142 (1)
		(0)				(3)					(1)	(1)	(1)

Table 2—continued

Sequence of experimental sessions, number of changeovers between keys. Shown in parentheses are the scheduled reinforcement rates on each key, with Keys 1, 2, 3, and 4 reading downward. The data shown are the sum, and below this in brackets the standard deviation, of the last five experimental sessions of the condition.

Condition (Rf/H)	Bird	Changeovers from One Key to Another Key											
		1 → 2	1 → 3	1 → 4	2 → 1	2 → 3	2 → 4	3 → 1	3 → 2	3 → 4	4 → 1	4 → 2	4 → 3
1	H1	111 (9)	271 (12)	0	382 (20)	328 (26)	0	0	599 (31)	0	0	0	0
(off)	H2	112 (11)	285 (23)	0	397 (34)	673 (52)	0	0	958 (43)	0	0	0	0
(30)	H3	449 (35)	110 (7)	0	559 (37)	557 (20)	0	0	667 (14)	0	0	0	0
(15)	H4	530 (61)	44 (1)	0	574 (61)	394 (37)	0	0	438 (38)	0	0	0	0
(off)	H5	101 (1)	51 (1)	0	152 (2)	1321 (49)	0	0	1372 (50)	0	0	0	0
	H6	350 (24)	71 (10)	0	421 (15)	473 (17)	0	0	544 (10)	0	0	0	0
2	H1	166 (4)	32 (3)	12 (2)	184 (4)	161 (11)	212 (7)	23 (2)	319 (9)	19 (2)	3 (1)	72 (3)	168 (5)
(7.5)	H2	296 (11)	44 (5)	1 (0)	182 (13)	506 (14)	10 (1)	59 (3)	140 (19)	359 (14)	40 (2)	322 (14)	8 (2)
(30)	H3	175 (14)	156 (10)	21 (3)	165 (10)	548 (24)	58 (7)	166 (20)	399 (16)	154 (9)	21 (2)	197 (14)	15 (3)
(15)	H4	270 (3)	15 (1)	0	213 (9)	285 (5)	179 (8)	67 (7)	238 (7)	7 (2)	5 (1)	169 (6)	12 (1)
(10)	H5	293 (24)	29 (3)	20 (3)	60 (6)	598 (12)	114 (9)	222 (27)	355 (31)	126 (11)	60 (7)	124 (5)	76 (4)
	H6	171 (6)	106 (7)	56 (11)	268 (14)	271 (20)	96 (5)	57 (5)	314 (13)	106 (5)	8 (1)	150 (9)	100 (7)
3	H1	166 (4)	32 (3)	12 (2)	184 (4)	161 (11)	212 (7)	23 (2)	319 (9)	19 (2)	3 (1)	72 (3)	168 (5)
(7.5)	H2	296 (11)	44 (5)	1 (0)	182 (13)	506 (14)	10 (1)	59 (3)	140 (19)	359 (14)	40 (2)	322 (14)	8 (2)
(30)	H3	175 (14)	156 (10)	21 (3)	165 (10)	548 (24)	58 (7)	166 (20)	399 (16)	154 (9)	21 (2)	197 (14)	15 (3)
(15)	H4	270 (3)	15 (1)	0	213 (9)	285 (5)	179 (8)	67 (7)	238 (7)	7 (2)	5 (1)	169 (6)	12 (1)
(10)	H5	293 (24)	29 (3)	20 (3)	60 (6)	598 (12)	114 (9)	222 (27)	355 (31)	126 (11)	60 (7)	124 (5)	76 (4)
	H6	171 (6)	106 (7)	56 (11)	268 (14)	271 (20)	96 (5)	57 (5)	314 (13)	106 (5)	8 (1)	150 (9)	100 (7)

Changeover data not measured

Table 2—continued

Changeovers from One Key to Another Key

Condition (Rf/H)	Bird	1 → 2	1 → 3	1 → 4	2 → 1	2 → 3	2 → 4	3 → 1	3 → 2	3 → 4	4 → 1	4 → 2	4 → 3
4													
(7.5)	H1	9 (3)	3 (1)	164 (11)	87 (7)	1 (0)	128 (4)	44 (2)	26 (2)	62 (4)	45 (6)	181 (7)	128 (3)
(30)	H2	45 (5)	8 (2)	93 (4)	33 (4)	130 (5)	324 (7)	23 (1)	2 (1)	167 (9)	90 (1)	440 (5)	54 (9)
(15)	H3	34 (3)	30 (2)	81 (7)	23 (3)	34 (1)	23 (2)	4 (0)	2 (1)	62 (3)	118 (10)	44 (3)	4 (1)
(60)	H4	35 (4)	1 (0)	90 (4)	48 (4)	0 (0)	327 (18)	13 (2)	33 (2)	49 (4)	65 (5)	307 (20)	94 (4)
	H5	7 (2)	2 (1)	188 (7)	52 (6)	53 (3)	243 (3)	58 (8)	15 (2)	152 (18)	87 (5)	326 (5)	170 (9)
	H6	2 (1)	3 (1)	165 (7)	140 (5)	13 (3)	162 (9)	7 (1)	46 (4)	53 (6)	23 (3)	267 (6)	90 (5)
5													
(7.5)	H1	20 (3)	29 (4)	202 (9)	178 (9)	95 (4)	280 (12)	50 (2)	203 (9)	18 (2)	23 (2)	330 (17)	147 (8)
(30)	H2	106 (5)	15 (2)	14 (3)	47 (2)	469 (5)	93 (8)	42 (1)	167 (13)	339 (9)	46 (1)	336 (4)	64 (4)
(15)	H3	345 (18)	33 (7)	10 (1)	161 (10)	432 (9)	57 (3)	68 (3)	56 (4)	357 (5)	159 (6)	249 (7)	16 (1)
(20)	H4	107 (12)	3 (1)	79 (3)	181 (12)	141 (3)	378 (12)	3 (1)	157 (4)	53 (6)	5 (1)	436 (13)	69 (5)
	H5	9 (1)	93 (6)	176 (11)	168 (6)	185 (10)	208 (9)	45 (3)	114 (9)	198 (10)	65 (3)	438 (8)	79 (4)
	H6	16 (2)	33 (5)	198 (4)	226 (4)	33 (4)	265 (8)	15 (2)	327 (13)	37 (4)	6 (1)	181 (7)	313 (13)
6													
(7.5)	H1	8 (3)	7 (2)	65 (1)	11 (2)	37 (9)	118 (5)	8 (1)	16 (1)	68 (5)	61 (4)	142 (9)	48 (5)
(30)	H2	54 (2)	21 (2)	9 (1)	10 (1)	34 (3)	248 (6)	7 (1)	39 (2)	65 (3)	67 (1)	199 (6)	56 (4)
(15)	H3	39 (5)	16 (1)	9 (2)	33 (3)	76 (4)	24 (2)	7 (1)	3 (1)	98 (5)	24 (2)	91 (1)	16 (2)
(120)	H4	77 (5)	5 (1)	21 (2)	61 (7)	16 (3)	151 (6)	17 (3)	21 (3)	15 (1)	25 (3)	130 (10)	32 (3)
	H5	45 (4)	17 (3)	15 (1)	44 (4)	68 (3)	51 (2)	0 (0)	17 (1)	123 (3)	33 (2)	101 (2)	55 (2)
	H6	2 (1)	1 (0)	95 (4)	76 (4)	1 (0)	113 (4)	5 (1)	72 (2)	20 (1)	17 (3)	116 (5)	95 (2)

Table 2—continued

Condition (Rft/H)		Changeovers from One Key to Another Key											
Bird		1 → 2	1 → 3	1 → 4	2 → 1	2 → 3	2 → 4	3 → 1	3 → 2	3 → 4	4 → 1	4 → 2	4 → 3
(7.5)	H1	391 (111)	173 (56)	71 (13)	504 (153)	578 (157)	341 (67)	92 (17)	765 (198)	85 (29)	39 (10)	267 (68)	191 (31)
	H2	302 (13)	181 (19)	38 (3)	473 (17)	333 (6)	161 (9)	33 (3)	481 (19)	73 (6)	15 (2)	184 (9)	73 (8)
	H3	213 (8)	89 (4)	1 (0)	187 (11)	346 (5)	2 (1)	89 (7)	194 (15)	160 (9)	27 (3)	128 (6)	8 (1)
	H4	160 (14)	13 (2)	36 (10)	206 (21)	351 (35)	147 (15)	1 (0)	354 (36)	17 (2)	2 (1)	190 (34)	8 (2)
	H5	157 (11)	40 (3)	17 (2)	161 (13)	421 (20)	198 (5)	19 (1)	457 (16)	71 (9)	34 (4)	166 (8)	86 (3)
	H6	48 (7)	272 (13)	49 (4)	327 (10)	174 (6)	122 (5)	35 (3)	389 (14)	54 (5)	7 (2)	186 (8)	32 (2)
(2xtn)	H1	2 (1)	1 (0)	0 (0)	3 (1)	438 (12)	282 (7)	0 (0)	559 (11)	11 (2)	0 (0)	162 (4)	131 (7)
	H2	58 (4)	13 (2)	0 (0)	31 (2)	726 (14)	218 (9)	26 (2)	611 (15)	104 (5)	14 (1)	306 (8)	2 (1)
	H3	12 (2)	1 (0)	0 (0)	10 (1)	772 (21)	40 (6)	1 (0)	460 (14)	341 (13)	2 (1)	350 (18)	29 (5)
	H4	2 (1)	0 (0)	0 (0)	1 (0)	417 (9)	337 (9)	1 (0)	503 (6)	12 (2)	0 (0)	250 (11)	99 (7)
	H5	30 (10)	6 (0)	1 (0)	25 (10)	749 (39)	130 (10)	4 (1)	661 (43)	132 (9)	8 (1)	213 (4)	42 (3)
	H6	1 (0)	10 (2)	3 (1)	11 (2)	536 (12)	168 (12)	3 (1)	565 (13)	143 (6)	0 (0)	149 (8)	165 (4)
	H1	0	0	0	0	275 (5)	94 (2)	0	320 (6)	128 (3)	0	49 (1)	173 (5)
	H2	15 (5)	2 (1)	0	2 (1)	464 (8)	6 (1)	12 (3)	140 (6)	332 (5)	3 (1)	317 (4)	18 (2)
	H3	0	0	0	0	470 (15)	4 (0)	0	440 (14)	148 (6)	0	34 (2)	118 (7)
	H4	0	0	0	0	401 (10)	131 (6)	0	391 (8)	49 (3)	0	141 (7)	39 (3)
	H5	0	0	0	0	393 (4)	0	0	326 (6)	125 (3)	0	67 (3)	58 (2)
	H6	0	0	0	0	296 (5)	35 (4)	0	321 (7)	118 (7)	0	10 (1)	143 (5)

Table 2—continued

Condition (Rft/H)	Bird	Changeovers from One Key to Another Key											
		1 → 2	1 → 3	1 → 4	2 → 1	2 → 3	2 → 4	3 → 1	3 → 2	3 → 4	4 → 1	4 → 2	4 → 3
10													
	H1	0 (0)	0 (0)	0 (0)	0 (0)	372 (7)	0 (0)	0 (0)	372 (7)	0 (0)	0 (0)	0 (0)	0 (0)
(Extn)	H2	21 (3)	0 (0)	0 (0)	20 (2)	499 (3)	5 (1)	1 (0)	493 (4)	5 (1)	0 (0)	10 (2)	0 (0)
(75)	H3	0 (0)	0 (0)	0 (0)	0 (0)	560 (6)	0 (0)	0 (0)	560 (6)	0 (0)	0 (0)	0 (0)	0 (0)
(37.5)	H4	0 (0)	0 (0)	0 (0)	0 (0)	384 (6)	3 (1)	0 (0)	387 (6)	9 (0)	0 (0)	0 (0)	3 (1)
(Extn)	H5	0 (0)	0 (0)	0 (0)	0 (0)	410 (12)	3 (1)	0 (0)	382 (12)	28 (1)	0 (0)	31 (1)	0 (0)
	H6	7 (1)	0 (0)	0 (0)	7 (1)	427 (2)	0 (0)	0 (0)	427 (2)	1 (0)	0 (0)	0 (0)	1 (0)
11													
	H1	0 (0)	0 (0)	0 (0)	0 (0)	88 (5)	5 (1)	0 (0)	90 (4)	0 (0)	0 (0)	3 (1)	2 (1)
(Extn)	H2	0 (0)	0 (0)	0 (0)	0 (0)	14 (2)	16 (3)	0 (0)	14 (2)	0 (0)	0 (0)	16 (3)	0 (0)
(112.5)	H3	1 (0)	0 (0)	0 (0)	1 (0)	20 (3)	0 (0)	0 (0)	20 (3)	0 (0)	0 (0)	0 (0)	0 (0)
(Extn)	H4	0 (0)	0 (0)	0 (0)	0 (0)	2 (1)	0 (0)	0 (0)	2 (1)	0 (0)	0 (0)	0 (0)	0 (0)
(Extn)	H5	11 (1)	0 (0)	0 (0)	11 (1)	2 (1)	1 (0)	0 (0)	2 (1)	0 (0)	0 (0)	1 (0)	0 (0)
	H6	0 (0)	0 (0)	0 (0)	0 (0)	17 (1)	4 (0)	0 (0)	20 (1)	0 (0)	0 (0)	1 (0)	3 (1)
12													
	H1	6 (2)	1 (0)	18 (1)	6 (1)	3 (1)	69 (2)	1 (0)	6 (1)	38 (4)	18 (1)	66 (3)	41 (4)
(7.5)	H2	1 (0)	0 (0)	0 (0)	1 (0)	19 (3)	209 (4)	0 (0)	9 (1)	38 (2)	0 (0)	219 (3)	28 (1)
(30)	H3	16 (3)	8 (1)	10 (2)	4 (1)	23 (3)	70 (3)	1 (0)	4 (1)	38 (3)	29 (1)	77 (5)	12 (1)
(15)	H4	2 (1)	0 (0)	3 (1)	0 (0)	16 (1)	100 (3)	0 (0)	30 (4)	7 (1)	5 (2)	84 (6)	21 (4)
(240)	H5	10 (1)	12 (1)	16 (2)	0 (0)	34 (3)	31 (1)	0 (0)	3 (1)	68 (2)	38 (2)	52 (3)	25 (2)
	H6	0 (0)	0 (0)	3 (1)	1 (0)	1 (0)	73 (1)	2 (1)	19 (3)	27 (2)	0 (0)	56 (2)	47 (2)

coded. The counter accumulating T_1 for the i th key timed from the first response on that key to the first response on another key. Therefore, in the present experiment, where ΣT = total session time,

$$\Sigma T = T_1 + T_2 + T_3 + T_4.$$

In all conditions but the first, the number of changeovers from a key i to another key j were recorded.

RESULTS

The responses, time, reinforcement, and changeover data used in the following graphs and analyses were obtained by summing the data from the final five sessions of each experimental condition. These data are presented for each bird in Table 2. The arithmetic sum of the individual data is designated the group data.

The numbers of responses on each key i were divided by the session time, ΣT , to obtain the response rate (P_i). The number of re-

inforcements obtained for pecks on each key was divided by ΣT to obtain the reinforcement rate (R_i). The number of changeovers (N_{ij}) from a key i to another key j was divided by ΣT to obtain the specific changeover rate (C_{ij}). The changeover rate (C_i) from a key i to all other keys j , k , and l was obtained from

$$C_i = C_{ij} + C_{ik} + C_{il}.$$

Response, time, reinforcement, and changeover data expressed in ratios were transformed according to the ratios A/B , B/C , C/D , and D/A , where A , B , C , and D are the data from Keys 1, 2, 3, and 4 respectively. These ratios use all the data without redundancy. The straight-line constants and exponents were calculated by the method of least squares when the data were in logarithmic form.

Figure 1 shows the ratios of response rates emitted (P_i/P_j) in the various conditions as a function of the ratio of the reinforcement rate (R_i/R_j) obtained for each bird. Equations for all the data are shown in each graph. In these

Table 3

Obtained values of constants c , k , and m , power a , intercept I , proportion of data variance accounted for by the regression fit v , and the number of datum points in each of the regression analyses n , for the individual data for Equations 1, 2, 4, 5, 14, 17, and 18.

Equation	Equation Number		Bird H1	Bird H2	Bird H3	Bird H4	Bird H5	Bird H6
$\frac{P_i}{P_j} = c \left[\frac{R_i}{R_j} \right]^a$	1	n	32	30	32	32	32	30
		c	1.02	1.03	1.03	1.06	1.05	1.03
		a	0.76	0.93	0.82	0.93	0.91	0.93
		v	0.90	0.87	0.90	0.88	0.81	0.93
$\frac{T_i}{T_j} = c \left[\frac{R_i}{R_j} \right]^a$	2	n	32	30	32	32	32	30
		c	1.02	1.01	1.02	1.06	0.99	1.04
		a	0.85	0.89	0.92	1.02	0.91	0.79
		v	0.95	0.88	0.92	0.91	0.91	0.92
$P_i = k \left[\frac{R_i}{\Sigma R} \right]^a$	4	n	45	45	45	45	45	45
		I	0.82	3.5	0.86	0.56	0.59	1.81
		k	45.6	65.3	73.2	72.0	68.8	70.6
		v	0.87	0.89	0.85	0.84	0.77	0.90
$P_i = k \frac{R_i^a}{\Sigma(R^a)}$	5	n	45	45	45	45	45	45
		I	-0.32	3.10	-0.28	0.15	0.04	0.27
		k	39.8	62.7	65.5	69.1	65.3	62.0
		v	0.90	0.89	0.86	0.84	0.77	0.92
$C_{ij} = m \frac{P_i P_j}{\Sigma P}$	14	n	63	63	63	63	63	63
		I	0.01	-0.09	-0.10	-0.07	0.10	-0.05
		m	0.30	0.25	0.23	0.23	0.20	0.19
		v	0.78	0.95	0.92	0.87	0.81	0.84
$C_{ij} = m \frac{T_i T_j}{(\Sigma T)^2}$	17	n	63	63	63	63	63	63
		I	-0.01	-0.08	-0.11	-0.03	-0.01	-0.04
		m	15.7	20.1	19.1	17.2	16.0	15.2
		v	0.82	0.95	0.84	0.91	0.85	0.92
$C_{ij} = m \frac{R_i R_j}{(\Sigma R)^2}$	18	n	63	63	63	63	63	63
		I	0.05	-0.02	-0.09	-0.05	0.00	0.01
		m	15.3	20.07	18.4	16.0	16.3	15.2
		v	0.82	0.85	0.78	0.78	0.85	0.90

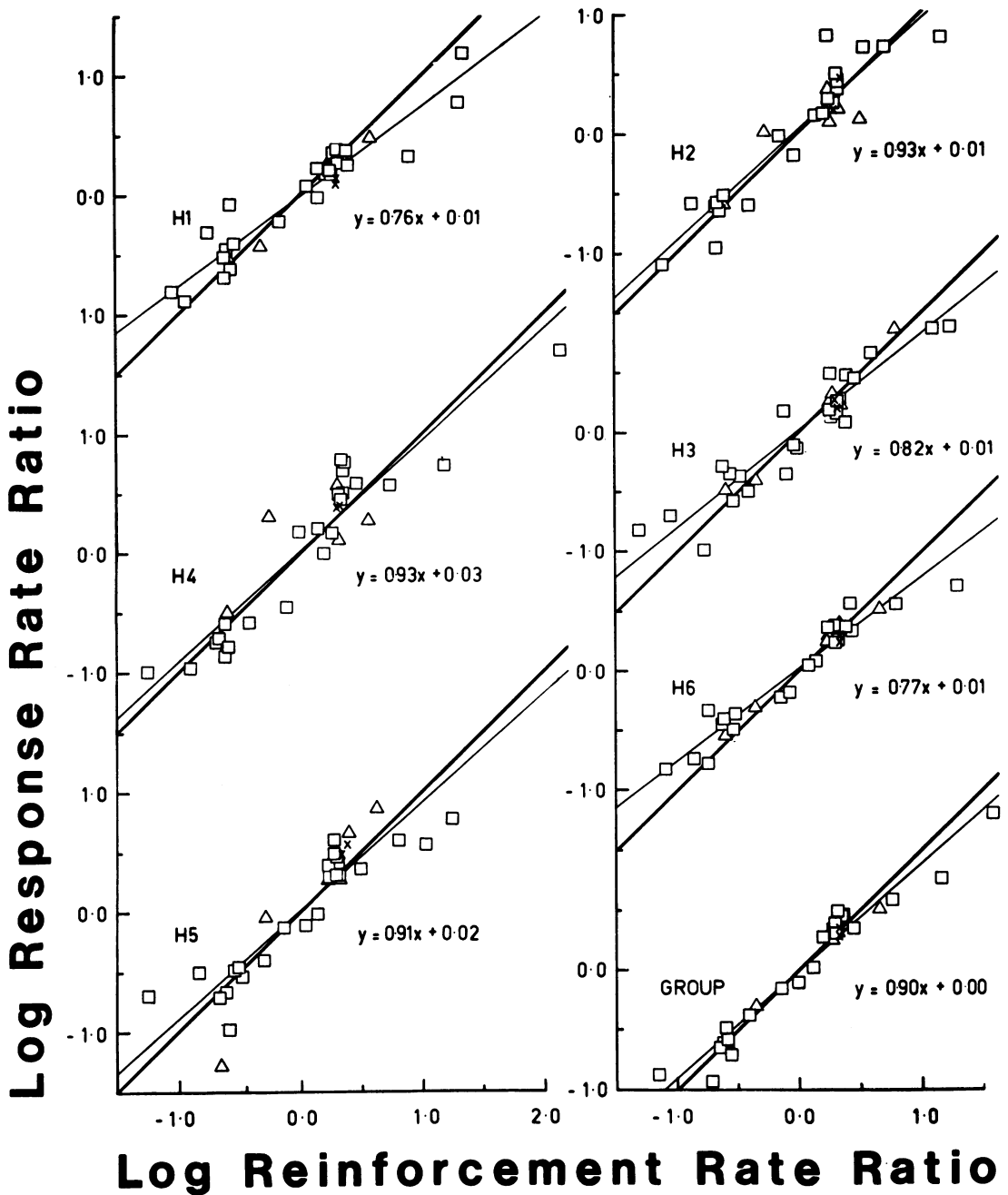


Fig. 1. The logarithm of the response-rate ratio ($\log P_1/P_1$) plotted against the logarithm of the reinforcement-rate ratio ($\log R_1/R_1$) for the individual bird and the group data. The number of keys in the condition from which a particular datum point is taken is indicated in the plots by \times , Δ , and \square for the 2-, 3-, and 4-key conditions respectively. The matching line (heavy), the regression line (fine), and the regression equation are also shown.

equations, the slope of the line is the exponent a_1 in Equation 1, and the constant is the logarithm of the multiplier c_1 .

Figure 1 and Table 3 show that the preference between the schedules was described by Equation 1 with a bias c_1 of close to 1.0 and

an exponent a_1 of between 0.76 and 0.93 (mean 0.85). In the analyses of individual bird performance (Table 3), the proportion of data variance accounted for by the fitted line was between 0.81 and 0.95 (mean 0.88). Table 3 shows that the response-rate exponent (a_1) was

always less than the time-allocation exponent (a_2) except for Bird H2.

The absolute response rates were analyzed according to Equations 4 and 5. Figure 2 shows the individual and group plots of the response rate on each key (P_i) as a function of the obtained reinforcement-rate proportion ($R_i/\Sigma R$)^a or $R_i^a/\Sigma(R^a)$. The values of the exponent a found from fitting Equation 1 to the data (Table 3) were used to calculate ($R_i/\Sigma R$)^a and $R_i^a/\Sigma(R^a)$. In the analyses of individual bird performance with respect to Equation 4 (Table 3), k (responses per minute) varied from 45.6 to 73.2 (mean 65.9), and proportion of data variance accounted for by the fitted lines varied from 0.77 to 0.90 (mean 0.85).

In the analysis of individual bird performances with respect to Equation 5 (Table 3), k (responses per minute) varied from 39.8 to 69.1 (mean 60.7), and proportion of data variance accounted for by the fitted lines varied from 0.77 to 0.92 (mean 0.86).

DISCUSSION

Preference

The data (Table 3) in this experiment fail to support the suggestion that response ratios match or equal reinforcement ratios (Herrnstein, 1970) or that time-allocation ratios match reinforcement ratios (Baum, 1974b; Baum and Rachlin, 1969). Rather, the individual and group data undermatch in both measures (Baum, 1974b; Davison and Hunter, 1976; Lobb and Davison, 1975; Myers and Myers, 1977). The proposition that the exponents in the time-allocation functions are larger than those in the response functions (Davison and Hunter, 1976; Lobb and Davison, 1975) is generally supported by the present results (Table 3).

Changeovers

Each time the subject makes a response, it chooses between n alternatives, and upon making a response is said to have made one of the n possible choices. It may choose to respond on the key it responded to last or may respond to one of the $n - 1$ alternative keys, effecting any of the $n - 1$ changeovers from the total of $n(n - 1)$ different types of changeovers.

If each successive choice is assumed to be independent of the last choice made, then the sequence of responses may be described by a

zeroth-order Markov chain (Chatfield and Lemon, 1970). It can be proved that the expected changeover rate (C_{ij}) in such a Markov process is given by

$$C_{ij} = \frac{P_i P_j}{\Sigma P} \quad (13)$$

Equation 13 expresses the changeover rate between two alternatives as a function of the product of the response rates on the two alternatives divided by the total response rate (ΣP). However, with the introduction of a changeover delay (COD), the assumption that each successive choice is independent of the last cannot be made, as subjects infrequently make a changeover during a COD (Catania, 1961). Because of this, introduction of a COD decreases the changeover rate (Catania, 1966).

One of the simplest generalizations of Equation 13 is that with a constant COD:

$$C_{ij} = m_1 \frac{P_i P_j}{\Sigma P} \quad (14)$$

In this equation, changeover rate (C_{ij}) is a direct function of the response rates. There are many other possibilities, but Equation 14 is theoretically appealing. For instance, Equation 14 implies that C_{ij} is maximal when P_i and P_j are equal. This result is consistent with reports (Baum, 1974a, 1976) that the changeover rates are highest when response rates on the two alternatives are equal. This result is not restricted to zero changeover delays. That is, m need not equal one.

Equation 14 satisfies the logical requirement of any changeover equation, that the changeover rate is zero (*i.e.*, $C_{ij} = 0$) when the response rate on one of the alternatives is zero (*i.e.*, $P_i = 0$). Herrnstein's (1961) difference relation (Equation 6) does not fulfill this requirement. Equation 14 also predicts that the changeover rate will increase with increases in the total response rate (ΣP).

If local response rates (N_i/T_i) are constant and equal, as they often nearly are in concurrent VI schedules (Catania, 1966), then it can be proved that Equation 14 becomes

$$C_{ij} = m_2 \frac{T_i T_j}{(\Sigma T)^2} \quad (15)$$

where $m_2 = m_1 N_i/T_i = m_1 N_j/T_j$.

Equation 15 predicts that in concurrent situations where the local response rates may be assumed constant and equal, changeover rate (C_{ij}) is a direct function of time allocation. Equation 15 predicts that the maximum

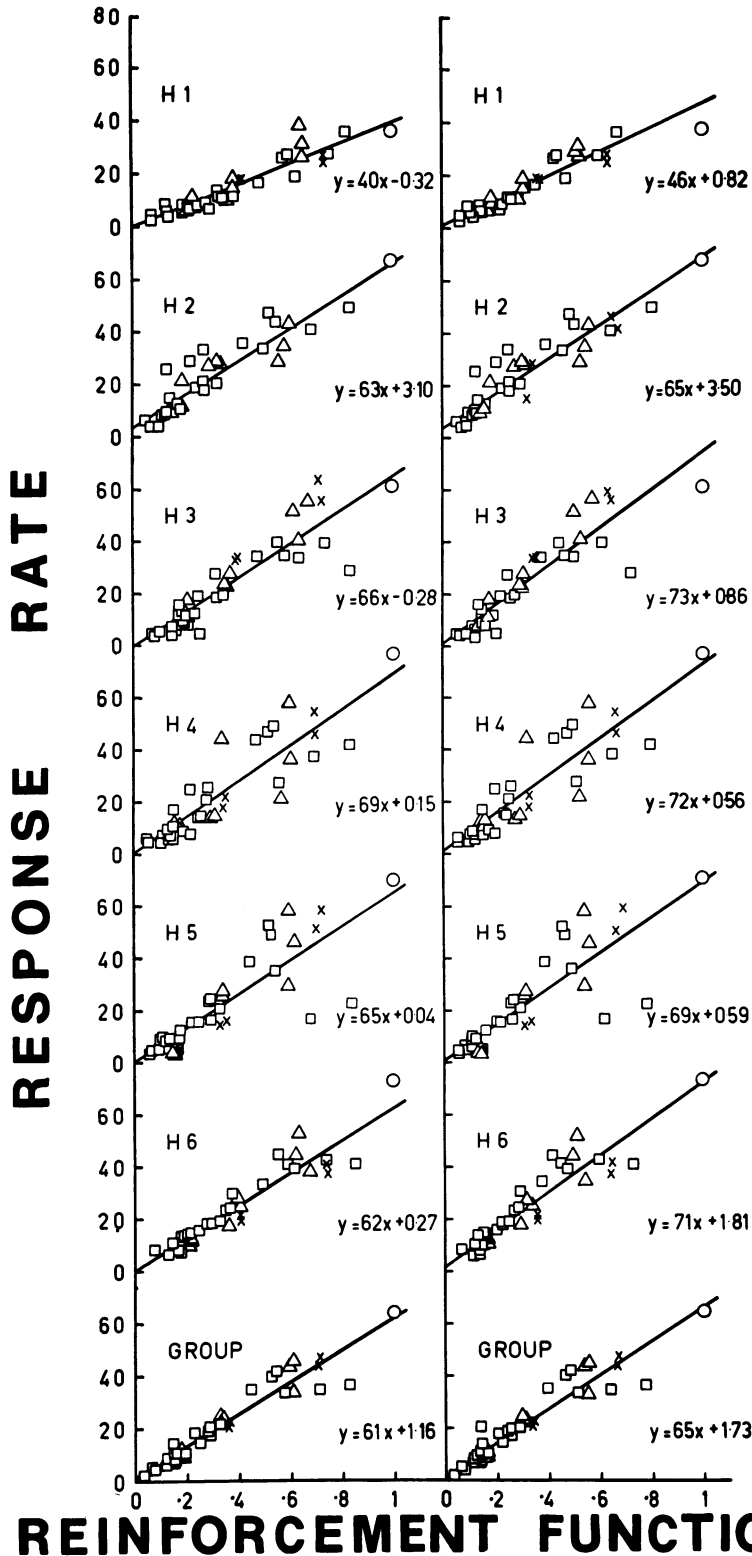


Fig. 2. The response rate on a key (P_1) plotted against the reinforcement-rate functions $R_1^*/(\Sigma R)^*$ (left side of the plot) and $R_1^*/\Sigma(R^*)$ (right side of the plot) for the individual bird and group data. The number of keys in the condition from which a particular datum point is taken is indicated in the plots by 0, x, Δ, and □ for the 1-, 2-, 3-, and 4-key conditions respectively. The regression line and the regression equation are also shown.

changeover rate will occur when the time is equally allocated between alternatives.

A similar equation can be derived for expressing changeover rate as a function of reinforcement rate. By substituting Equation 4 or 5 into Equation 14, when the exponent $a = 1$ and the total response rate k are constant, gives

$$C_{ij} = m_3 \frac{R_i R_j}{(\Sigma R)^2}, \quad (16)$$

where $m_3 = m_1 k$. If a does not equal one in Equations 4 and 5, various expressions can be derived, depending on which equations (4a,b, 5a,b) for total output (ΣP) are used. The relation used by Stubbs, Pliskoff, and Reid (1977) is given by Equation 4 in combination with Equation 4b.

Given the various assumptions stated, Equation 16 predicts that the changeover rate will be maximal when the reinforcement rates are equal. Equation 16 also predicts that the changeover rate (C_{ij}) is zero when the reinforcement (R_i) for one alternative is zero. This means that Equation 16 is not as powerful as Equations 14 or 15, because these latter equations can account for changeovers when responding is occurring on an alternative that is not providing reinforcement. There are many experimental manipulations (*e.g.*, time-outs, changes in reinforcement magnitude, *etc.*) which, although affecting changeover rate, will not significantly affect the reinforcement rate. In such situations, Equation 16 unlike Equations 14 and 15 will fail to predict changes in the changeover rate (C_{ij}). In Figure 3, changeover rate (C_{ij}) is plotted against the response-rate function (Equation 14), the time-allocation function (Equation 15), and the reinforcement function (Equation 16) for individual and group data. The least-squares best-fit lines are shown together with the corresponding equation. In the analysis of individual bird performance shown in Table 3, the proportion of data variance accounted for by the fitted lines is high: for Equation 14, it varied from 0.78 to 0.95 (mean 0.86); Equation 15, 0.82 to 0.95 (mean 0.88); Equation 16, 0.78 to 0.90 (mean 0.83).

In view of the success of Equations 14, 15, and 16 in accounting for the present changeover data, it is of importance to assess their application to a wide range of concurrent VI experiments in the literature.

The changeover data from a wide range of experiments have been analyzed according to Equations 14, 15, and 16 (the results of these analyses are available from the authors). The relationship between changeover rate and response rate (Equation 14), and changeover rate and time allocation (Equation 15), and where applicable changeover rate and reinforcement rate (Equation 16), holds not only when frequency of reinforcement is varied on two keys (*e.g.*, Herrnstein, 1961; Schmitt, 1974), three keys (Pliskoff and Brown, 1976; Reynolds, 1963), and five keys (Miller and Loveland, 1974), but also when the frequency of punishment imposed is varied (Deluty, 1966), delay of reinforcement varied (Cicerone, 1976), per cent deprivation varied (McSweeney, 1975), magnitude of shock varied (Todorov, 1971), and concurrent chain schedules varied (Baum, 1974; MacEwen, 1972). When the data from these experiments were plotted on logarithmic coordinates, the regression equation determined by a least-squares fit usually accounted for more than 90% of the data variance.

These results suggest that many of the variables that affect changeover rate also affect response rate and time allocation, as expressed in Equations 14 and 15, by the same amount. One variable that does alter this relationship is the changeover delay (COD).

Brownstein and Pliskoff (1968) reported that changeover rate decreases as COD is increased. When changeover rate (C_{ij}) was plotted as a function of $P_i P_j / \Sigma P$ for a series of COD values, a family of linear functions result. Each line in the family has a different slope (m), which corresponds to a particular COD value. The relation between the slopes (m) of these lines and COD is well described by the following equation

$$\log \frac{1}{m} = -e(\log \text{COD}) - \log b. \quad (17)$$

In some experiments (*e.g.*, Pliskoff, 1971), the COD from alternatives i to j is not equal to COD from j to i . Pliskoff (1971) had differences as much as $\text{COD}_{ij} = 27$ sec, $\text{COD}_{ji} = 3$ sec.

Equation 17 must be extended to account for such COD asymmetry by the addition of the COD values in the logarithmic form. Equation 17 expands to

$$\log \frac{1}{m} = -e[\log \text{COD}_{ij} + \log \text{COD}_{ji}] - \log b. \quad (18)$$

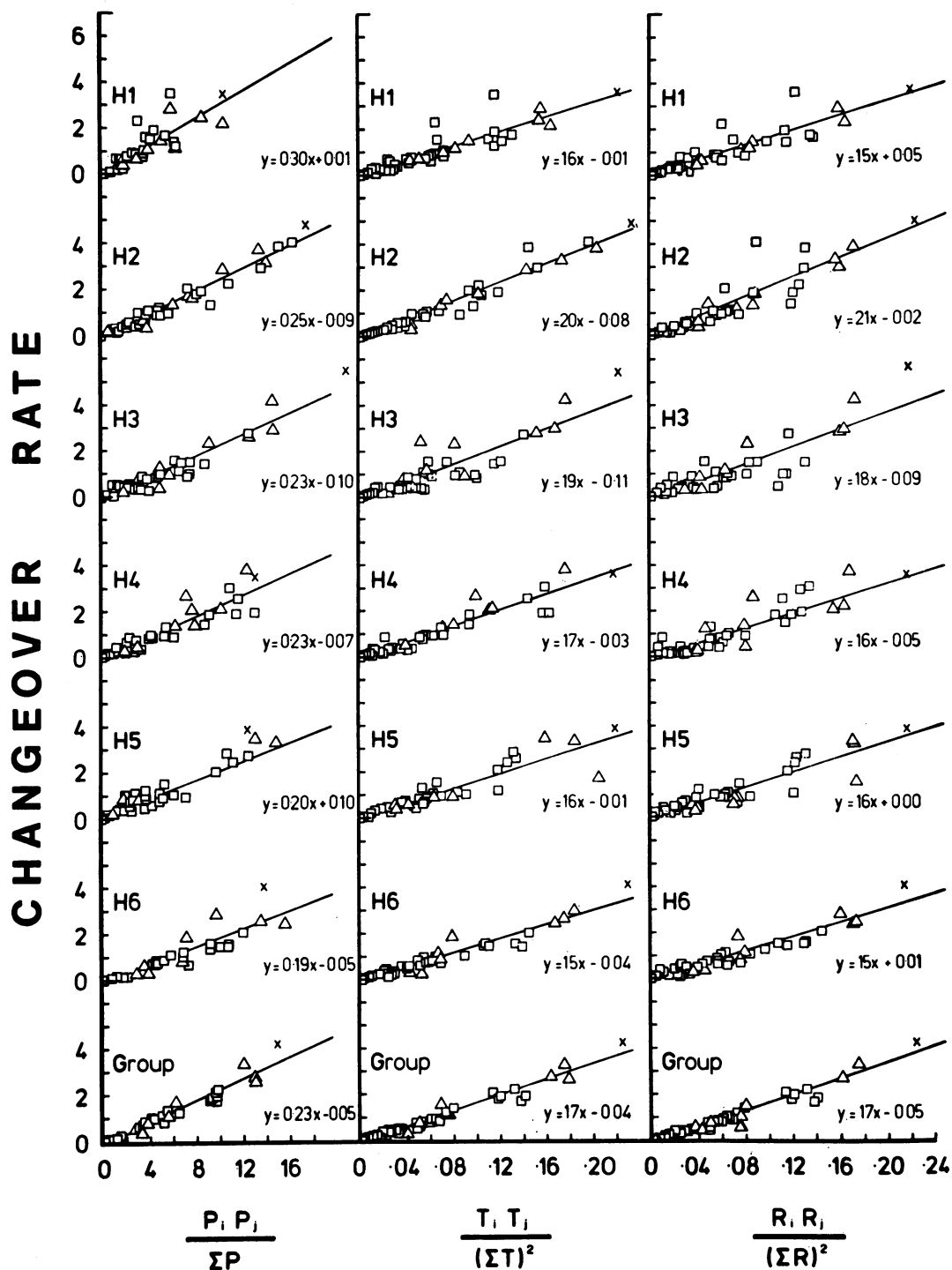


Fig. 3. The changeover rate (C_{1j}) plotted against the three changeover functions $P_j P_j / \Sigma P$, $T_j T_j / (\Sigma T)^2$, and $R_j R_j / (\Sigma R)^2$ for the individual and group data. The number of keys in the condition from which a particular datum point is taken is indicated in the plots by \times , Δ , and \square for the 2-, 3-, and 4-key conditions respectively. The regression line and the regression equation are also shown.

With zero COD values, Equations 17 and 18 become invalid. A factor G may be added to the COD values to avoid such a result. G might reflect the fact that even without a COD, a small amount of time is still required to move from one key to the other, or to peck the changeover key. This suggestion is consistent with the obtained values of G , which are in the order of 1 or 2 sec.

Aside from the zero COD reason for adding a constant G to the COD value, its addition to Equation 18 increases the data variance. With G added, Equation 18 becomes

$$m = b[(\text{COD}_{ij} + G)(\text{COD}_{ji} + G)]^e. \quad (19)$$

The value of G in Equation 19 may be obtained by iteration to give a maximum correlation between the values of the left- and right-hand sides of the equation.

Equation 19 may now be substituted for m in Equations 14, 15, and 16 to give

$$C_{ij} = b_1 [(\text{COD}_{ij} + G)(\text{COD}_{ji} + G)]^e \frac{P_i P_j}{\Sigma P} \quad (20)$$

$$C_{ij} = b_2 [(\text{COD}_{ij} + G)(\text{COD}_{ji} + G)]^e \frac{T_i T_j}{(\Sigma T)^2} \quad (21)$$

$$C_{ij} = b_3 [(\text{COD}_{ij} + G)(\text{COD}_{ji} + G)]^e \frac{R_i R_j}{(\Sigma R)^2}. \quad (22)$$

Equations 20, 21, and 22 enable prediction of changeover rate when the changeover delays are also varied.

In Figure 4, the changeover-rate data from some experiments (Brownstein and Pliskoff, 1969; Brownstein and Shull, 1970; Pliskoff, 1971; Schroeder, 1975; Schroeder and Holland, 1969; Shull and Pliskoff, 1967; Stubbs and Pliskoff, 1969) in the literature, where COD has been varied in concurrent VI schedules, are plotted as a function of the response-rate formula (Equation 20) or, if those data were not available, the time-allocation formula (Equation 21). Where possible, data from individuals rather than the group data are presented. The species of the subject and the subject's initials or number, together with the line and equation of the least-squares fit, are recorded on Figure 4.

The changeover-rate data from the experiments depicted in Figure 4 were analyzed according to the response-rate function (Equation 20), time-allocation function (Equation 21), and the reinforcement-rate function (Equation 22). In Table 4, the constants and the least-squares fit correlation (r) resulting

from such analyses are presented. Individual subjects and other experiments from the literature were omitted from Table 4 if the number of data points available was less than four. Other omissions from Table 4 occurred when either response-rate, time-allocation, or reinforcement-rate data were not available. The correlations were typically above 0.9, showing that the Equations 20, 21, and 22 account for a large proportion of the data variance. With increases in the COD, the changeover rate decreases because the subjects remain in an alternative longer. Other procedures can also affect the average time a subject spends responding to an alternative before changing to another. For instance, the introduction of fixed-interval second-order components into a concurrent VI schedule might be expected to have similar effects to the introduction of CODs.

Cohen (1976) reported such an experiment where the FI components were varied asymmetrically. When the scheduled FI values were substituted for CODs in Equations 20, 21, and 22, the changeover rates (C_{ij}) were predicted by these equations. The correlations (r) and parameters e , b , and G for the iterative analysis and least-squares fits for Cohen's (1976) data as described by Equations 20, 21, and 22 are presented in Table 4. Once again, the correlations were high.

In summary, the relationship between changeover rate and response rate (Equation 14) or changeover rate and time allocation (Equation 15) appears to remain relatively invariant as the values of a number of independent variables (such as reinforcement rate, food deprivation, *etc.*) are changed. The functions are, however, affected by procedures that cause the subject to alter its changeover rate while the response rates remain essentially unchanged. Two such procedures are introduction of a COD or introduction of second-order FI components. The way in which the relationship changes with variation in the COD or second-order component values appears to be regular and accountable for by regular changes in the slope (m) of the functions. These regular changes in the slope are themselves accounted for by nonlinear functions (Equations 20, 21, and 22) of either the COD values or second-order FI component values.

The relationships between changeover rate and response rate or time allocation do not re-

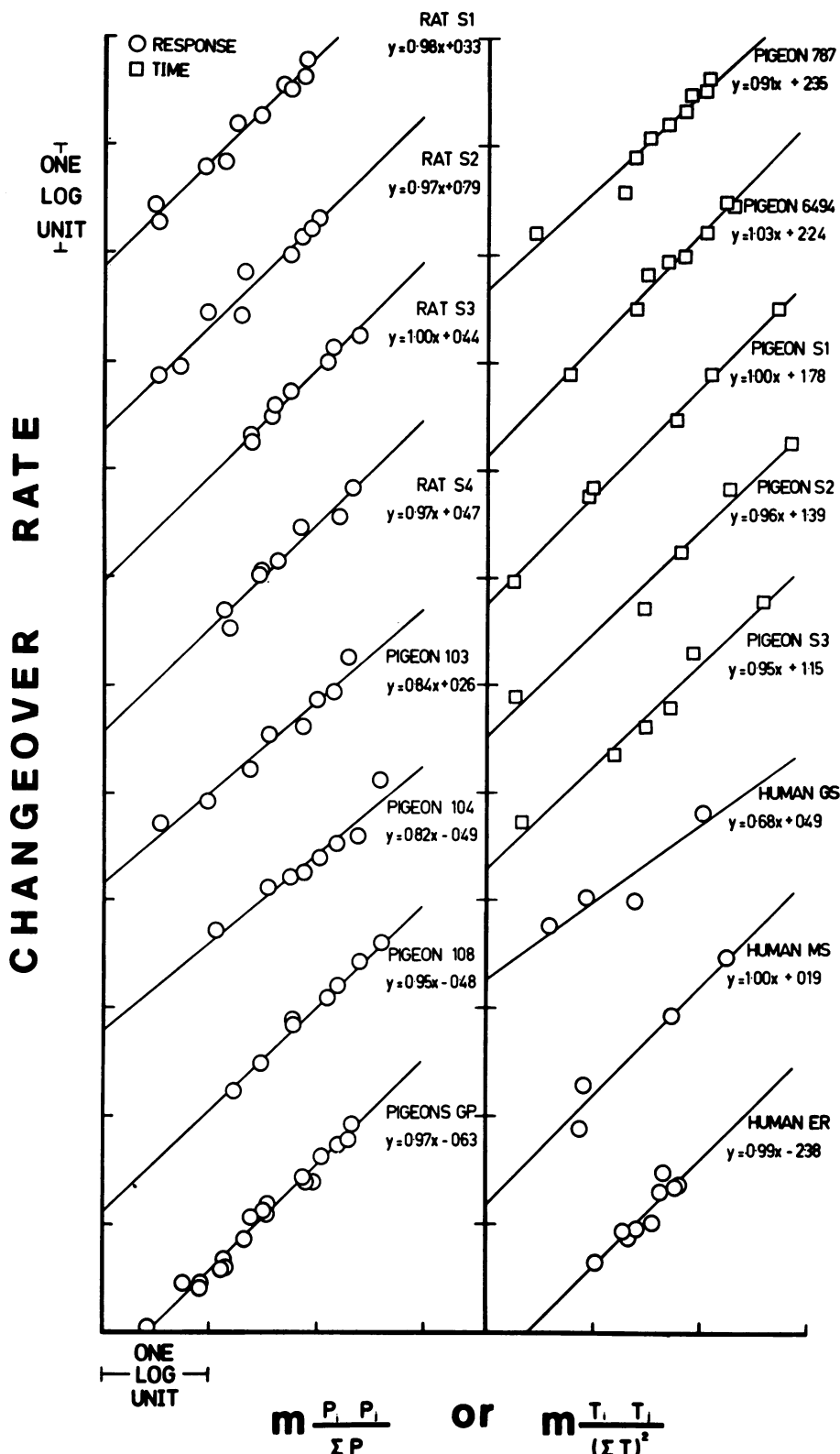


Fig. 4. The changeover rate (C_{11}) plotted against the changeover function $mP_1P_1/\Sigma P$ (circles) or where these were not available the changeover function $mT_1T_1/(\Sigma T)^2$ (squares). The subject's name and species, the regression line, and the regression equation are also shown. Both axes are logarithmic.

Table 4

The analysis of the changeover rate (C_{12}) data as a function of the response-rate (P), time-allocation (T) or reinforcement-rate (R) data from a range of experiments where the changeover delays (COD) were varied. The number of datum points (n) used in the least-squares analysis and the resultant slope (b), and correlation (r) are shown for each subject, together with the constants G and e, which were obtained by reiteration to maximize the correlation (r).

Report	Subject and Species*	$C_{12} = b_1[(COD_{12} + G)(COD_{21} + G)]^{\frac{P_1 P_2}{\Sigma P}}$					$C_{12} = b_2[(COD_{12} + G)(COD_{21} + G)]^{\frac{T_1 T_2}{(\Sigma T)^2}}$					$C_{12} = b_3[(COD_{12} + G)(COD_{21} + G)]^{\frac{R_1 R_2}{(\Sigma R)^2}}$				
		r	n	G	e	b ₁	r	n	G	e	b ₂	r	n	G	e	b ₃
Schroeder, 1975	MR (H)	0.92	9	2.3	-0.28	0.01	0.93	9	1.9	-0.25	0.47	0.90	9	2	-0.18	0.58
Pliskoff, 1971	Group (P)	0.99	19	0.50	-0.41	2.3	0.99	19	1.25	-0.55	26.5	0.96	19	1.50	-0.64	39.1
Brownstein & Shull, 1970	S1 (P)						1.00	6	1.30	-0.64	59.8	1.00	6	0.65	-0.52	30.8
	S2 (P)						0.99	5	1.15	-0.51	24.4	1.00	5	1.65	-0.73	55.6
	S3 (P)						0.98	6	0.45	-0.36	14.0	0.99	6	0.25	-0.30	12.2
Schroeder & Holland, 1969	MS (H)	0.97	4	1.5	-0.55	1.53										
	GS (H)	0.92	4	2.0	-0.66	3.09										
	RM (H)	0.94	3	3.0	-1.26	23.8										
	SR (H)	0.97	3	3.0	-0.58	3.96										
Stubbs & Pliskoff, 1969	103 (P)	0.90	9	2.0	-0.74	1.65	0.96	9	2.0	-0.57	28.8	0.96	8	2.0	-0.61	59.9
	104 (P)	0.92	9	0.8	-0.32	0.45	0.96	9	0.8	-0.37	6.95	0.95	8	0.8	-0.35	13.1
	108 (P)	1.00	9	0.2	-0.34	0.33	0.99	9	0.2	-0.27	13.7	0.95	8	0.2	-0.32	9.59
Brownstein & Pliskoff, 1969	787 (P)						0.97	9	3.8	-0.72	221.8	0.99	8	3.8	-0.80	521.3
	6494 (P)						0.99	8	2.3	-0.81	174.5	0.98	7	2.3	-0.86	227.9
Shull & Pliskoff, 1967	S1 (R)	0.99	10	2.7	-0.64	2.12	0.98	10	2.7	-0.56	25.4	0.99	10	2.7	-0.56	30.7
	S2 (R)	0.98	9	3.3	-0.87	6.14	0.99	9	3.3	-0.68	64.9	0.99	9	3.3	-0.70	82.5
	S3 (R)	0.99	8	4.0	-0.67	2.73	0.99	8	4.0	-0.61	59.4	0.99	8	4.0	-0.65	52.3
	S4 (R)	0.97	8	3.0	-0.74	2.96	0.97	8	3.0	-0.67	42.2	0.97	8	3.0	-0.67	42.2
Cohen, 1975	58 (P)	0.98	5	1.5	-0.22	0.40	0.94	5	1.5	-0.30	47.2	0.89	5	1.5	-0.23	34.3
	59 (P)	0.98	5	0.2	-0.43	1.04	0.99	5	0.2	-0.39	46.0	0.99	5	0.2	-0.46	69.4
	60 (P)	0.98	5	4.8	-0.65	8.33	0.96	5	4.8	-0.65	224.3	0.98	5	4.8	-0.60	226.3
	61 (P)	0.97	5	0.3	-0.52	1.58	0.98	5	0.3	-0.43	64.0	0.96	5	0.3	-0.47	66.3
	62 (P)	0.99	5	0.6	-0.38	0.94	1.00	5	0.6	-0.36	37.1	0.92	5	0.6	-0.42	39.5
	63 (P)	0.97	5	0.6	-0.60	4.36	0.96	5	0.6	-0.40	79.8	0.91	5	0.6	-0.44	81.7

*H = human, P = pigeon, R = rat.

quire knowledge of the reinforcement rate. Thus, testing such relationships in natural environments where no explicit reinforcement rate can be defined for the observed behaviors should prove interesting.

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